Formalizing Perfect Information Extensive Form Games

Game Theory Course:
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Introduction

- The normal form game representation does not incorporate any notion of sequence, or time, of the actions of the players
- The extensive form is an alternative representation that makes the temporal structure explicit.
- Two variants:
  - perfect information extensive-form games
  - imperfect-information extensive-form games

Definition

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- Players: N is a set of n players

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- Players: N
- Actions: A is a (single) set of actions

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- Players: N
- Actions: A
- Choice nodes and labels for these nodes:
  - Choice nodes: H is a set of non-terminal choice nodes

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  - Action function: $\chi : H \mapsto 2^A$ assigns to each choice node a set of possible actions

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- Players: N
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  - Player function: $\rho : H \mapsto N$ assigns to each non-terminal node h a player $i \in N$ who chooses an action at h

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- Choice nodes and labels for these nodes:
  - Choice nodes: H
  - Action function: $\chi : H \mapsto 2^A$
  - Player function: $\rho : H \mapsto N$
- Terminal nodes: Z is a set of terminal nodes, disjoint from H

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  - Choice nodes: H
  - Action function: $\chi : H \mapsto 2^A$
  - Player function: $\rho : H \mapsto N$
- Terminal nodes: Z
- Successor function: $\sigma : H \times A \mapsto H \cup Z$ maps a choice node and an action to a new choice node or terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$
  - Choice nodes form a tree: nodes encode history

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  - Choice nodes: H
  - Action function: $\chi : H \mapsto 2^A$
  - Player function: $\rho : H \mapsto N$
- Terminal nodes: Z
- Successor function: $\sigma : H \times A \mapsto H \cup Z$
- Utility function: $u = (u_1, \ldots, u_n)$; $u_i : Z \mapsto \mathbb{R}$ is a utility function for player i on the terminal nodes Z

Example: the sharing game